Analysis and Design Considerations for Pretensioned Cable Vehicle Restraint Systems

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ABSTRACT

Pretensioned cables restraint systems are commonly used around the perimeter of parking garages. Typical building code requirements treat the design of such systems as a static load problem placing a requirement on the total lateral force the restraint system must provide. In fact, the restraint system is intended to safely bring a slowly rolling vehicle to a to stop after initial contact. The governing vehicle/cable restraint dynamics are presented as a nonlinear second-order equation of motion and numerical solutions of the equation of motion are obtained for realistic design scenarios to evaluate the cable deflection, increase in cable tension, and the vehicle velocity envisioned by the Building Code.

INTRODUCTION

Pretensioned cables are frequently used around the perimeter of parking garages as a vehicular restraint system. The 2009 edition of the International Building Code (IBC) [Ref. (a)] requires restraint systems be designed for a 6,000-lbf lateral force. The Code does not define a corresponding deflection limit nor does it define a design impact scenario (vehicle mass and velocity at first contact). Parking garages occasionally have precast panels or face brick that are not designed to resist vehicle-induced lateral forces. The restraint system cannot allow vehicles to impact these non-structural elements. Additionally, the increase in cable tension that occurs is significant and must also be accounted for in the design of columns and stanchions that support the cables.

GOVERNING EQUATION FOR VEHICLE IMPACT WITH A CABLE RESTRAINT SYSTEM

The cable restraint system provides a restoring force that opposes the vehicle's motion that is a function of the initial pretension force and the increase in cable force caused by elongation. The projection of that net force along the vehicle's direction of motion is the restoring force. A typical impact scenario is depicted in Figure 1. The columns are spaced at a distance L and the vehicle (mass, M and initial velocity, V) contacts the cables a distance rL from one of the columns, where $0 < r < 1$.

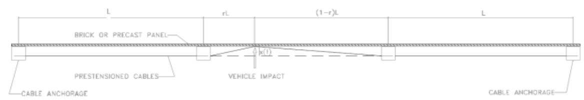

Figure 1 Vehicle Impact Scenario

The cable elongation and projection of the cable tension along the vehicle's path depends upon the cable deflection $x(t)$. The elongation, ΔL, is determined from

$$\begin{aligned}\Delta L &= \sqrt{x^2 + (rL)^2} + \sqrt{x^2 + (1-r)^2 L^2} - L \\ \Delta L &\approx \frac{x^2}{2L}\left(\frac{1}{r} + \frac{1}{1-r}\right) = \frac{x^2}{2L}\left(\frac{1}{r - r^2}\right) = \frac{x^2 b(r)}{2L}\end{aligned} \tag{1}$$

where the first two terms in the Taylor expansion of the square root are used. Equation (1) is valid for $0.1 < r < 0.9$, which is more than adequate for the present analysis. The design condition of most interest occurs when the vehicle contacts the cable at midspan [$r = 0.5$, $b(0.5) = 4$].

The tension in the cables, $T(x)$, as a function of cable deflection and initial pretension force in all cables, T_0, is given by

$$T(x) = \frac{\Delta L(x)}{nL} EA + T_0 \tag{2}$$

The cable length between anchorages is nL, where n is the integer number of bays that the cable spans. E is Young's modulus for the cables and A is the total area of all cables that participate in providing the restoring force.

The projection of the cable tension force normal to the cable (*i.e.*, the worst-case impact scenario), $F(x)$, is computed from

$$F(x) = -T(x)\frac{xb(r)}{L}$$
$$T(x) = \frac{EAb(r)}{2nL^2}x^2 + T_0 \quad (3)$$

The impulse-momentum principle provides the connection between the normal force and vehicle momentum while in contact with the cables. The normal force acting on the car for a short period of time, Δt, is the impulse, which is equivalent to the change in momentum of the vehicle:

$$F(x)\Delta t = \Delta(Mv) = M\Delta\dot{x}$$
$$F(x) = M\frac{\Delta\dot{x}}{\Delta t} \rightarrow M\ddot{x} \quad (4)$$

Substituting Equations (1) through (3) into Equation (4) leads to a second-order, nonlinear ordinary differential equation with initial conditions.

$$M\ddot{x} + \frac{EAb(r)^2}{2nL^3}x^3 + \frac{T_0 b(r)}{L}x = 0 \quad (5)$$
$$x(0) = 0, \quad \dot{x}(0) = V$$

where the cubic nonlinearity arises from the large deflection of the cable, while the pretension force is responsible for the linear term. Typically, structural deformation is small and the nonlinear terms are insignificant. In this case, a cable deflection of 12 to 20 in. over a 27-ft span results in a nonlinear force comparable in magnitude to the linear force. Hence, the nonlinear force cannot be ignored in this application and, in fact, provides a significant portion of the total restoring force.

The governing differential equation assumes that the columns supporting the cables are rigid, all of the cables participate equally in restraining the vehicle, friction between the columns and cables is negligible, and the vehicle is rolling freely with no rolling resistance or applied engine-generated power. The equation provides the cable deflection as a function of time, *t*, beginning from first contact. The maximum deflection when the vehicle's velocity drops to 0 from *V* is of primary interest. The time at which this occurs is of no importance in the present investigation.

APPLICATION TO THE BUILDING CODE REQUIREMENT

The IBC requires the vehicle restraint system be designed to resist a force, F_{IBC}, of 6,000 lbf. This requirement is interpreted as a static load requirement as there is no mention of a nominal vehicle mass or initial velocity. In this case, the governing equation becomes

$$F = \frac{EAb(r)^2}{2nL^3}x^3 + \frac{T_0 b(r)}{L}x \quad (6)$$

which can be used to determine the cable deflection, *x*, required to meet the Code-defined resistance. The increase in cable tension, $T(x)$, can then be determined from the initial tension and the lateral deflection.

The cable deflection might be used to ensure that the vehicle does not contact a non-structural element (*e.g.*, a masonry curtain wall or precast side panel) before achieving the Code-mandated resistance. Likewise, the cable tension (at that deflection) is used to verify the cable anchorage structure can withstand the increased lateral force.

Consider a typical design scenario where 6 1/2-in. diameter cables are assumed to engage a vehicle. More cables might be specified in a given cable restraint system; however, only those cables that contact and deflect with the vehicle should be considered. Each cable is pretensioned to 3 kips. The corresponding parameters for use in Equation (6) are the total cable area A = 0.918 in^2, initial cable force T_0 = 18 kips, and E = 29,000 ksi. Typical unsupported cable spans (L) range between 20 to 30 ft. The overall cable length between anchorages will be some multiple of the cable span, but is typically limited to one or two bay lengths.

A plot of the restoring force versus cable deflection in shown in Figure 2 for a 28-ft bay width and three different cable lengths (*i.e.*, spans between anchorages). A line representing the IBC lateral force requirement is shown in the figure for reference. The cable deflection associated with the IBC force varies from about 9 in. to 12 in., depending upon the anchorage spacing. A longer cable allows more deflection than does a shorter cable.

Architectural features not designed to resist lateral forces should be located with sufficient clearance to allow for the cable deflection.

Quasi-structural elements (*e.g.*, precast panels or unreinforced masonry walls) may be placed closer to the cables if they are designed to resist that portion of the IBC-mandated force not carried by the cables when contact occurs. In the example above, 28-ft-long cables, anchored at 28 ft on center may be installed 4 in. from a masonry wall if the wall is designed to withstand a force of 4,750 lbf because the cables only provide 1,250 lbf with 4 in. of deflection.

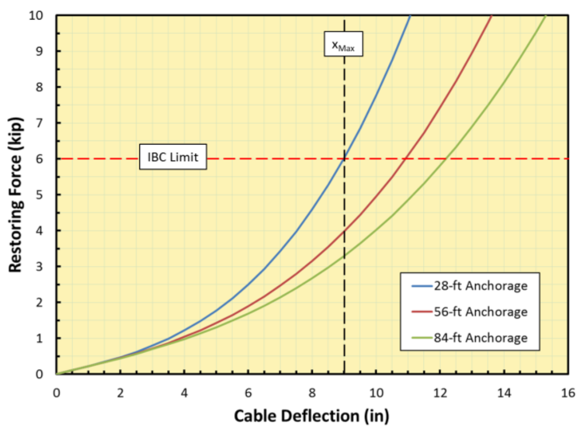

Figure 2 Cable Restoring Force VS Deflection

The restoring force produced by the cable restraint system also produces an increase in the cable tension force that the cable support structure must be designed to resist. Cable tension, for the 6 cables engaged with the vehicle, is plotted in Figure 3 for the three cable lengths considered in this example.

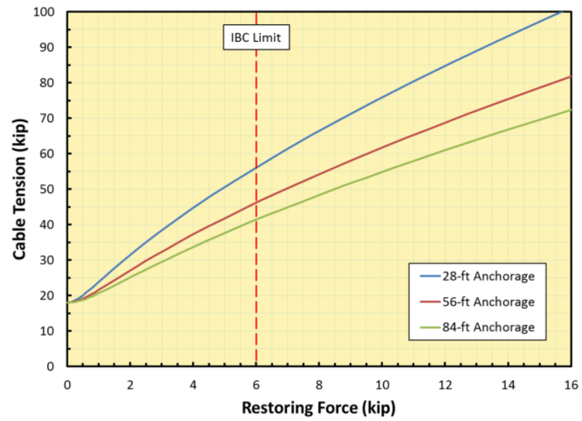

Figure 3 Cable Tension VS Restoring Force

The initial total tension force is 18 kips for the 6 cables. The combination of cable deflection and stretching causes this force to increase to 40 to 55 kips, depending upon cable length. The remaining cables in the restraint system experience no increase in their initial tension and this force should be added to the tension obtained from Figure 3. Assuming, say, 4 additional cables are employed in the restraint system, an additional 12 kips must be added to the 55 kips (for 28 ft between anchorages) for a total tension of 67 kips.

Equation (6) may also be used to determine the pretension force required to produce a 6,000-lbf restoring force for a prescribed displacement:

$$T_0 = \frac{F_{IBC} L}{b(r)x} - \frac{EAb(r)x^2}{2nL^2} \qquad (7)$$

The total pretension force required for 6 28-ft long cables to develop the IBC force with, say, 4 in. deflection is 120 kip (20 kip/cable). Of course, all of the cables in the restraint system must be tensioned to this forces because there is no way of knowing which of the 6 cables will participate in the restraining force. A 10-cable system would impose a 200-kip force on the cable anchorage structure, which may be impractical.

APPLICATION TO STOPPING A ROLLING VEHICLE

Bringing a slowly rolling vehicle to a stop is the most likely scenario requiring a 6,000-lbf restraining force, but the Code does not define the parameters for such a scenario (vehicle mass and initial velocity). The maximum required restoring force may be used to determine the corresponding maximum initial velocity of a nominal-weight vehicle (about 4,500 lbf) to gain some insight into the impact scenario envisioned by the Code.

As a vehicle contacts the cable restraint system, the elongation and cables produces the restoring force that brings the vehicle to a stop. The cable tension increases during this interaction as a consequence of the elongation. This force increases to a maximum as the vehicle is brought to a stop.

Cable deflection and induced tensile force required to stop a rolling vehicle are investigated here for several impact scenarios. A vehicle is assumed to contact six (6) 28-ft-long restraint cables at midspan. The cables are assumed to be anchored at each end of their 28-ft length. A plot of the vehicle velocity versus cable deflection is shown in Figure 4 for different initial rolling vehicle speeds ranging from 1 mph to 10 mph. In each case, the resulting cable deflection curve is relatively flat, which implies the cables provide very little resistance immediately after first contact.

A car that impacts the cable restraint system at 1 mph will be brought to a stop in 5.3 in., whereas 18.7 in. of lateral cable displacement is required to stop a car initially moving at 10 mph.

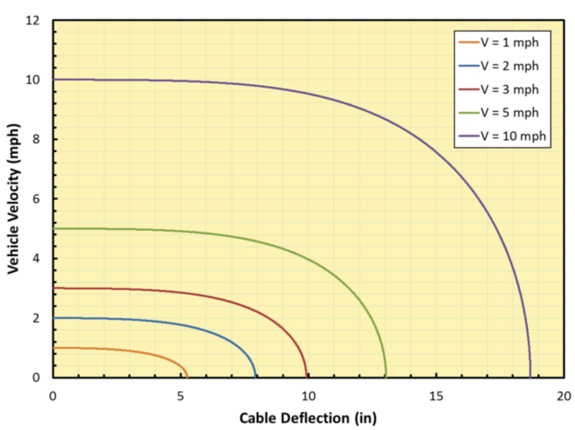

Figure 4 Vehicle Velocity VS Cable Deflection

The corresponding restoring force can be determined from the cable deflection and Figure 2, using the "28-ft Anchorage" curve. The cable tension force can then be obtained from Figure 3 from the restoring force. The maximum values for cable deflection (x_{Max} in inches), restoring force (F_{Max} in kips), and cable tension (T_{Max} in kips) when the vehicle is brought to a stop are summarized in Table 1 for the five impact velocities investigated here.

Table 1 Maximum Cable Deflection and Forces

	1 mph	2 mph	3 mph	5 mph	10 mph
x_{Max}	5.3"	7.9"	9.9"	13"	18.7"
F_{Max}	1.9 k	4.5 k	7.6 k	15.2 k	40.6 k
T_{Max}	31 k	48 k	64 k	98 k	183 k

The IBC requires the vehicle restraint system be designed to resist 6,000 lbf, which corresponds to a vehicle impact velocity of about 2.5 mph and a required cable deflection of about 9 in. for this typical cable geometry (28-ft-long cables anchored at each end). The cable restraint support structure (columns and/or stanchions) where the cables are anchored must be designed to resist lateral forces corresponding to the maximum cable tension force, or about 60 to 70 kips (accounting for the additional cables that do not participate in stopping the vehicle).

A typical cable restraint system designed to satisfy the IBC-prescribed force can be overwhelmed relatively easily. The ultimate tensile force a single cable can withstand is about 40 kips. For six cables, the total maximum tension force is 240 kips. A vehicle that impacts the restraint system at a velocity greater than about 14 mph will cause the cables to snap before the vehicle is brought to a stop.

CONCLUSIONS

The design of a typical vehicle restraint system composed of pretensioned cables is deceptively more complex than the Building Code implies. The IBC treats restraint system design as a statics problem. In fact, the governing equation must account for the dynamic interaction among the nonlinear cable stiffness, the mass of the vehicle, the initial velocity, and vehicle deceleration. The analyses performed here suggest the Code requirement corresponds roughly to bringing a 4,500-lbf vehicle to a stop after contacting the restraint system at an initial velocity of 2.5 mph.

Additional complications arise when the cables run parallel to a non-structural element such as an unreinforced masonry wall. Care must be taken in these cases to ensure the cables develop the Code-required 6,000-lbf lateral force before the cables contact the wall. Very little restoring force is generated immediately following contact. A typical design scenario studied here suggests that a minimum of 9 in. between the cables and the nonstructural element is required for the cables to develop the Code-required lateral force.

Elastic stretching of the cables is the primary mechanism responsible for the restoring force. Hence, the cable restraint support structure must be designed for the increase in cable tension that results while generating the lateral force. Cables are typically pretensioned to 3 kips each at installation. When the cables are deflected to provide the required restoring force, the tension force in each cable increases three-fold, to about 9 kips per active cable. Hence, the supports of a 10-cable restraint system, with 6 cables actively resisting the impact must be designed for a total lateral force of 67 kips rather than the initial total pretension force of 30 kips.

REFERENCES

(a) “2009 International Building Code,” International Code Council, February 2009.